

PENTAQUARK WITH DIQUARK CORRELATIONS IN A QUARK MODEL

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We have investigated $uudd\bar{s}$ pentaquarks by employing quark models with the meson exchange and the effective gluon exchange as qq and $q\bar{q}$ interactions. The system for five quarks is dynamically solved; two quarks are allowed to have a diquark-like qq correlation. It is found that the lowest mass of the pentaquark is about 1947 – 2144 MeV. There are parameter sets where the mass of the lowest positive-parity state become lower than that of the negative-parity states. Which parity corresponds to the observed peak is still an open question. Relative distance of two quarks with the attractive interaction is found to be by about 1.2 – 1.3 times closer than that of the repulsive one. The diquark-like quark correlation seems to play an important role in the pentaquark systems.

1. Introduction

Since the experimental discovery of the baryon resonance with strangeness +1, $\Theta(1540)^+$,¹ many attempts have been performed to describe the peak theoretically.² To describe this resonance by using a quark model, one needs at least five quarks, $uudd\bar{s}$, which is called a pentaquark. A quark model, however, seems to have difficulties to explain some of the features of this peak. Namely, (1) the observed mass is rather low, (2) the observed width is very narrow, and (3) there is only one peak is found, especially no $T=1$ peak nearby. To reproduce the observed mass, 100 MeV above the KN threshold, it is preferable to take lowest-mass configuration, $TJ^P=0\frac{1}{2}^-$. It is reported, however, that this state will have a very large width, which contradicts to the observed narrow width.³ Other candidate, $0\frac{1}{2}^+$, may have a rather narrow width, but it might not become as low as the observed one.

The fact that the peak is buried in the NK continuum makes the problem more difficult. As was reported in this workshop,^{4,5} the QCD lattice

calculation as well as the QCD sum rule approach found this continuum problem rather serious. The quark model can deal with this problem by introducing the scattering states using the resonating group method. Also, it is reported⁶ that a 'bound state' calculated without such scattering states becomes a resonance at almost the same energy when the scattering states are introduced though the levels which couple strongly to the nucleon-kaon systems disappear. In this work, we investigate the pentaquark systems without taking its breaking effect as a first step. Our main aim here is to investigate the effects of the qq correlation on the pentaquarks, which have been mainly treated only by a simple wave function.

We employ several parameter sets for the hamiltonian: those with the one-boson exchange (OBE), those with the one-gluon exchange (OGE), and those with the semirelativistic or the nonrelativistic kinetic energy terms. By employing these various parameter sets, we try to estimate the mass of pentaquarks of various quantum numbers with a controlled ambiguity.

2. Model

The hamiltonian for quarks and anti-quarks is taken as:

$$H_q = \sum_i K_i + v_0 + \sum_{i < j} (V_{\text{OGE}ij} + V_{\text{PS}ij} + V_{\sigma ij} + V_{\text{conf}ij}) \quad (1)$$

with

$$K_i = \sqrt{m_i^2 + p_i^2} \quad (\text{semi-rela}) \quad \text{or} \quad m_i + p_i^2/2m_i \quad (\text{non-rela}) . \quad (2)$$

The two-body potential term consists of the one-gluon-exchange potential, V_{OGE} , the one-PS-meson-exchange potential, V_{PS} , the one- σ -meson-exchange potential, V_{σ} , and the confinement potential, V_{conf} :

$$V_{\text{OGE}ij} = (\lambda_i \cdot \lambda_j) \frac{\alpha_s}{4} \left\{ \left(\frac{1}{r_{ij}} - \frac{e^{-\Lambda_g r_{ij}}}{r_{ij}} \right) - \left(\frac{\pi}{2m_i^2} + \frac{\pi}{2m_j^2} + \frac{2\pi}{3m_i m_j} (\sigma_i \cdot \sigma_j) \right) \frac{\Lambda_g^2}{4\pi} \frac{e^{-\Lambda_g r_{ij}}}{r_{ij}} \right\} \quad (3)$$

$$V_{\text{PS}ij} = \frac{1}{3} \frac{g^2}{4\pi} \frac{m_m^2}{4m_i m_j} (f_i \cdot f_j) (\sigma_i \cdot \sigma_j) \left\{ \frac{e^{-m_m r_{ij}}}{r_{ij}} - \left(\frac{\Lambda_m}{m_m} \right)^2 \frac{e^{-\Lambda_m r_{ij}}}{r_{ij}} \right\} \quad (4)$$

$$V_{\sigma ij} = -\frac{g_s^2}{4\pi} \left\{ \frac{e^{-m_m r_{ij}}}{r_{ij}} - \frac{e^{-\Lambda_m r_{ij}}}{r_{ij}} \right\} \quad (5)$$

$$V_{\text{conf}ij}^{(\lambda)} = -(\lambda_i \cdot \lambda_j) a_{\text{conf}} r_{ij} . \quad (6)$$

Table 1. Five parameter sets. Each parameter set is denoted by R π , or Rg π , etc., according to the kinetic energy term and the qq interaction. Each meson mass is taken to be the observed one, and $m_\sigma = 675$ MeV.

Model ID	qq int.	m_u [MeV]	m_s [MeV]	α_s	Λ_g [fm $^{-1}$]	$g_8^2/4\pi$	$(g_0/g_8)^2$	Λ_0 [fm $^{-1}$]	κ	a_{conf} [MeV/fm]	V_0 [MeV]
R π^\dagger	$\pi \sigma \eta$	313	530	0	-	0.69	0	1.81	0.92	170	-378.3
Rg π	OGE $\pi \sigma \eta \eta'$	340	560	0.35	3	0.69	1	1.81	0.92	172.4	-381.7
Ng π^\ddagger	OGE $\pi \sigma \eta$	313	550	0.35	5	0.592	0	2.87	0.81	172.4	-453
Ng	OGE	313	680	1.72	3	0	-	-	-	172.4	-345.5
Graz §	$\pi \eta \eta'$	340	500	0	-	0.67	1.34	2.87	0.81	172.4	-416

† Ref. 9, ‡ Ref. 8, and § Ref. 7.

Table 2. S -wave and P -wave quark pairs

	T	S	C	$(-)^{\ell}$		T	S	C	$(-)^{\ell}$
α	0	0	$\bar{\mathbf{3}}$	+	$\tilde{\alpha}$	0	0	$\mathbf{6}$	-
β	0	1	$\mathbf{6}$	+	$\tilde{\beta}$	0	1	$\bar{\mathbf{3}}$	-
γ	1	0	$\mathbf{6}$	+	$\tilde{\gamma}$	1	0	$\bar{\mathbf{3}}$	-
δ	1	1	$\bar{\mathbf{3}}$	+	$\tilde{\delta}$	1	1	$\mathbf{6}$	-

In V_{OGE} , α_s is the OGE strength, and Λ_g is the gluon form factor. In V_{PS} , g is the quark-meson coupling constant: $g = g_8$ for π , K , and η , and $g = g_0$ for η' meson. The value of g_8 can be obtained from the observed nucleon-pion coupling constant, $g_{\pi\text{NN}}$.^{7,8} The term proportional to Λ_m^2 is originally the δ -function term; the form factor for the meson exchange, Λ_m , is assumed to depend on the meson mass m_m as $\Lambda_m = \Lambda_0 + \kappa m_m$.^{7,8,9}

As for the confinement potential for the five quark system, we replace the factor $(\lambda_i \cdot \lambda_j)$ in eq. (6) by its average value as

$$V_{\text{conf}ij}^{(1)} = \frac{4}{3} a_{\text{conf}} r_{ij} . \quad (7)$$

This modified confinement potential gives the same value for the orbital $(0s)^5$ state as that of the original one. This replacement enables us to remove all the scattering states and to investigate only tightly bound states, which will appear as narrow peaks. After the coupling to the scattering states with an original confinement potential is introduced, some of the states we find will melt away into the continuum. The situation can be clarified by evaluating the width, which we will investigate elsewhere.

We have employed four kinds of parameter sets: R π , Rg π , Ng π and Ng (Table 1). R stands for the parameter sets with the semirelativistic kinetic energy term, while N stands for those with the nonrelativistic one. The parameter sets with OBE [OGE] are denoted by the name with π [g]. We also perform the calculation with the parameter set given by Graz group.⁷

The wave function we employ is written as:

$$\psi_{TSL}(\xi_A, \xi_B, \eta, \mathbf{R}) = \sum_{i,j,n,m,\omega,\omega',\lambda} c_{ijnm}^{\omega\omega'\lambda} \mathcal{A}_{q^4} \times \phi_{q^2}(\omega, \xi_A; u_i) \phi_{q^2}(\omega', \xi_B; u_j) \psi(\lambda, \eta; v_n) \Big|_L \chi_{\bar{s}}(\mathbf{R}; w_m), \quad (8)$$

where \mathcal{A}_{q^4} is the antisymmetrization operator over the four ud-quarks, and ξ_A, ξ_B, η and \mathbf{R} are the internal coordinates defined as:

$$\xi_A = \mathbf{r}_1 - \mathbf{r}_2 \quad \text{and} \quad \xi_B = \mathbf{r}_3 - \mathbf{r}_4, \quad (9)$$

$$\eta = (\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4)/2, \quad (10)$$

$$\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)/4 - \mathbf{r}_{\bar{s}}. \quad (11)$$

$\phi_{q^2}(\omega, \xi; u)$ is the wave function for two quarks with quantum number ω , which is one of $\alpha, \beta, \dots, \tilde{\delta}$ listed in Table 2, with size parameter u :

$$\phi_{q^2}(\omega, \xi; u) = \left\{ \begin{array}{l} \varphi([11]_{STC}) \\ \varphi([2]_{STC}) \end{array} \right\} \xi \exp \left[-\frac{\xi^2}{4u^2} \right] \left\{ \begin{array}{l} (\ell = 0) \\ (\ell = 1) \end{array} \right\}. \quad (12)$$

The relative wave function between two qq pairs, $\psi(\lambda, \eta; v)$ and the wave function between four quarks and \bar{s} quark, $\chi_{\bar{s}}(\mathbf{R}; w)$, are taken as:

$$\psi(\lambda, \eta; v) = \left\{ \begin{array}{l} 1 \\ \eta \end{array} \right\} \exp \left[-\frac{\eta^2}{2v^2} \right] \left\{ \begin{array}{l} (\lambda = 0) \\ (\lambda = 1) \end{array} \right\} \quad (13)$$

$$\chi_{\bar{s}}(\mathbf{R}; w) = \exp \left[-\frac{2R^2}{5w^2} \right]. \quad (14)$$

For the negative-parity pentaquarks ($L=0$), we use all possible $\ell=\ell'=\lambda=0$ states. For the positive-parity pentaquarks ($L=1$), we use the states where one of ℓ, ℓ' and λ is equal to 1. The gaussian expansions are taken as geometrical series: $u_{i+1}/u_i = v_{n+1}/v_n = w_{m+1}/w_m = 2$. We take 6 points for u (0.035–1.12 or 0.04–1.28fm), 4 points for v (0.1–0.8fm), and 3 points for w (0.2–0.8fm). Since we use a variational method, the obtained masses are the upper-limit. They, however, converge rapidly; the mass may reduce more, but probably only by several MeV.

3. Results and discussions

The mass of the $q\bar{q}$, q^3 , and $q^4\bar{q}$ systems are shown in Table 3. Parts of these baryon masses were given in refs. 8 and 9.

It is very difficult for a constituent quark model to describe the Goldstone bosons. Also, it is hard to justify the models with the kaon-exchange interaction between quarks to describe a kaon. We use the K^* mass as a reference for the pentaquarks.

Table 3. Masses of meson, baryon, and pentaquark, $P(TJ^P)$, in MeV.

	N	Δ	K*	NK*	P($0\frac{1}{2}^-$)		P($0\frac{3}{2}^-$)		P($1\frac{1}{2}^-$)		P($0\frac{1}{2}^+$) [‡]	
					m_P	m'_P	m_P	m'_P	m_P	m'_P	m_P	m'_P
R π	941	1261	979	1921	2109	2054	2141	2083	2143	2083	2165	2045
Rg π	938	1232	931	1869	1985	1947	2064	2018	2078	2021	2129	2006
Ng π	936	1232	814	1846	2029	1996	2106	2066	2106	2061	2321	2144
Ng	938	1232	814	1846	1966	1971	2153	2144	2170	2145	2345	2197
Graz	937	1239	927	1864	2231	2160	2240	2168	2251	2173	2248	2120
Exp. [†]	939	1232	892	1831	1540							

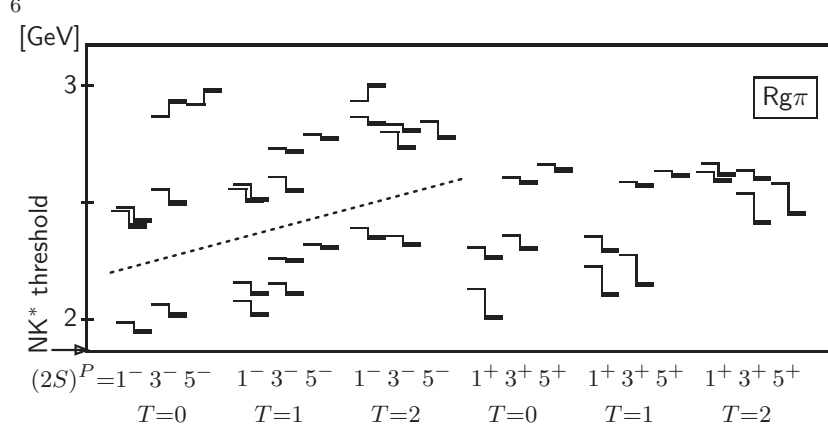
[†]Ref. 12. [‡]Since the interaction we use is central, $0\frac{1}{2}^+$ and $0\frac{3}{2}^+$ are degenerated.

Contrary to the $q\bar{q}$ systems, we have more satisfactory results for the q^3 baryons. The mass spectrum of the S -wave ground states is well reproduced. Moreover, since the chiral quark models have a mechanism to lower the mass of the Roper resonance than that of the negative-parity excited nucleons, the excited baryon mass spectrum can also be reproduced. On the other hand, in the OGE quark model picture, the Roper mass is considered to reduce by introducing the pion-cloud effect, which should be taken into account separately. Though it is interesting to see whether the Roper resonance has a pentaquark component,^{10,11} that is out of scope of our present work. Parameter sets Rg π and Ng underestimate the negative-parity excited nucleon mass by about 70 MeV and 90 MeV, respectively. When we discuss the positive-parity pentaquarks by these models, we will have to take this underestimate into account.

Now we discuss the system of the pentaquarks. There is no bound state when we use the original confinement, eq. (6). In Table 3, we show the masses of the pentaquarks with $V_{\text{conf}}^{(1)}$, m_P , and the mass with the correction from the confinement potential evaluated by the wave function corresponding to m_P :

$$m'_P = m_P + \langle V_{\text{conf}_{ij}}^{(\lambda)} \rangle - \langle V_{\text{conf}_{ij}}^{(1)} \rangle. \quad (15)$$

In Figure 1, we plot these m_P (thin bars) and m'_P (thick bars) for Rg π . Among the q^4 S -wave systems, five spin-isospin states can couple to the orbital $(0s)^4$ configuration: $(TS)=(01)$, (10) , (11) , (12) , and (21) .¹³ The negative-parity pentaquarks which have a large component of these q^4 states correspond to the levels under the dotted line in Figure 1. The mass difference between them and other states is about several hundred MeV. Among them, the $(TS) = (01)$ and (10) states are the lowest two states, which are essentially degenerated: *i.e.*, $(TS)J^P=(01)\frac{1}{2}^-$, $(01)\frac{3}{2}^-$, and $(10)\frac{1}{2}^-$. As OGE becomes stronger, $(01)\frac{1}{2}^-$ goes down. For example,

Figure 1. Pentaquark mass spectrum obtained by the $Rg\pi$ parameter set.

the splitting between $(01)\frac{1}{2}^-$ and $\frac{3}{2}^-$ is 71 MeV in $Rg\pi$ whereas it is 32 MeV in $R\pi$, or 9 MeV in $Graz$. It becomes 174 MeV in Ng , where all the hyperfine splitting comes from OGE. The remaining two levels, however, still stay close to each other.

There is not such a large separation in the mass spectrum of the positive parity pentaquarks, reflecting the fact that all of the spin-isospin states can couple to the orbital $(0s)^4 0p$ state. The interaction, however, makes one of the states very low: *i.e.*, $(00)\frac{1}{2}^+$ (and $\frac{3}{2}^+$ because the interaction is central in the present work). It can actually be as low as the negative-parity states.

The masses are still much higher than the observed peak, and depend on the parameters. There is a large ambiguity in the zero-point energy¹⁴ as well as in the interaction. For example, we should include the instanton induced interaction,¹⁵ which is the source of the η - η' mass difference. It seems, however, that the relative positions of the levels do not change much. We argue that one of the above mentioned levels is observed as the peak.

Because the $TJ^P=0\frac{1}{2}^-$ and $1\frac{1}{2}^-$ states couple to the NK state strongly and $1\frac{3}{2}^-$ couples to ΔK , it is unlikely that they appear as narrow peaks. It seems that $0\frac{3}{2}^-$ is a good candidate for the observed peak.¹⁴ Unfortunately, which of the above $0\frac{3}{2}^-$ and $0\frac{1}{2}^+$ states is most likely seen is still an open question. The $0\frac{3}{2}^-$ is lower than the other in $Ng\pi$ and Ng , and also in the $Rg\pi$ parameter set if its underestimate of the P -wave baryon mass is taking into account. On the other hand, the $0\frac{1}{2}^+$ state is lower than the other in the semirelativistic chiral models: $R\pi$ and $Graz$. In all the cases, however, the mass difference between these two states is not large.

Except for the confinement force, all the interaction terms are short-ranged in the quark model. Thus, when the two-quark correlation is intro-

Table 4. Values of $\langle \mathcal{O}_{FS} \rangle$ for each q^4 state.

	(T, S)	$\langle \mathcal{O}_{FS} \rangle$	diff	Full (R π) [MeV]
Parity $-$	(01),(10)	$-10 \ (-16)$		2131
	(11)	$-6 \ (-12)$	$\rangle \ 4 \ (4)$	2231 $\rangle \ 100$
	(12),(21)	$2 \ (-4)$	$\rangle \ 8 \ (8)$	2418 $\rangle \ 187$
Parity $+$	(00)	$-30 \ (-30)$		2178
	(11)	$-22 \ (-22)$	$\rangle \ 8 \ (8)$	2342 $\rangle \ 164$
	(01) ,(10)	$-10 \ (-16)$	$\rangle \ 12 \ (6)$	2465 $\rangle \ 123$

Table 5. Size of the quark pairs of each $(T_2 S_2)$ spin-isospin state in the $(TS)J^P = (01)\frac{1}{2}^-$ and $(00)\frac{1}{2}^+$ pentaquarks as well as that in the nucleon.

$(TS)J^P$	$(T_2 S_2)$ pair (R π)				$(T_2 S_2)$ pair (Rg π)			
	(00)	(01)	(10)	(11)	(00)	(01)	(10)	(11)
(01)1/2 $-$	0.53	0.70	0.68	0.62	0.56	0.69	0.68	0.64
(00)1/2 $+$	0.56	0.69	0.69	0.61	0.61	0.74	0.74	0.68
N	0.50	0.65	0.65	0.56	0.55	0.76	0.76	0.62

duced in the model, quark pairs where the interaction is attractive come closer while those with repulsion tend to stay apart from each other. Then an attractive pair may behave like a single particle, called a diquark.

In the chiral quark model with a simple gaussian wave function, the matrix element of the spin-isospin operator, $\langle \mathcal{O}_{FS} \rangle$, is proportional to the hyperfine splitting (Table 4):

$$\langle \mathcal{O}_{FS} \rangle = -\langle [f]_{TS} | \sum_{i < j} (\tau_i \cdot \tau_j)(\sigma_i \cdot \sigma_j) | [f]_{TS} \rangle. \quad (16)$$

On the other hand, suppose one takes a diquark-model picture, only the pairs between which the interaction is attractive have to be considered. The expectation values of $\langle \mathcal{O}_{FS} \rangle$ in this picture are listed in the parentheses in Table 4 alongside of the original matrix elements. The ‘mass difference’ between these states is also shown in the column under ‘diff’. The ratio of the mass differences of the positive-parity states is 8/12 in the shell-model picture while it is 8/6 in the diquark-like picture. The ratio obtained from the averaged masses by our full calculation is found to support the diquark-like picture. The qq correlation plays an important role in the pentaquarks.

More direct approach to see the importance of the qq correlation is to look into the size of the quark pairs, $\sqrt{\langle r^2 \rangle}$. In Table 5 we show the size of quark pair for each $(T_2 S_2)$ state in the lowest pentaquarks. Their size is large when the interaction is repulsive while it becomes small for the attractive pairs. The ratio is about 1.2 – 1.3. The degree of the qq correlation in the pentaquarks seems similar to that in the nucleon.

4. Summary

We have investigated $uudd\bar{s}$ pentaquarks by employing quark models with the meson exchange and the effective gluon exchange as qq and $q\bar{q}$ interactions. The system for five quarks is dynamically solved; two quarks are allowed to have a diquark-like qq correlation.

The present work indicates that the $TJ^P=0\frac{3}{2}^-, 0\frac{1}{2}^+$ pentaquark states can be almost as low as the $0\frac{1}{2}^-$ state, which has been assigned to the observed peak, but expected to have a large width. Both of the $0\frac{3}{2}^-$ and $0\frac{1}{2}^+$ states are considered to have a narrow width. Which parity should correspond to the observed peak is still an open question. Relative distance of two quarks with the attractive interaction is found to be by about 1.2 – 1.3 times closer than that of the repulsive one. The diquark-like quark correlation seems to play an important role in the pentaquark systems. The $1\frac{1}{2}^-$ state is also found to be low. Like the $0\frac{1}{2}^-$ state, however, it couples to the NK states strongly. This may be the reason why there is no peak in the $T=1$ channel.

As for the absolute mass, our estimate is still more than 400 MeV higher than the observed one. We consider the extra attraction may come from other qq interactions as well as from the ambiguous zero-point energy. The width and the resonant energy should be investigated by including the coupling to the baryon meson asymptotic states, which is underway.

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